Math 202 - Quiz I --- Sample 005

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1) Use **(Gauss) Divergence Theorem** carefully to find the outward **flux** $\iint_S \boldsymbol{F.n}\ \boldsymbol{d\sigma}$ of the vector field $F = 1/\rho\ < 9x,\ y,\ 5z >$ where $\rho = \sqrt{x^2 + y^2 + z^2}$ across

i) the surface $\rho = 2$

ii) the surface surrounding the region $1 \le \rho \le 2$ between two spheres.

2) Let **F = < 5y, 8y+2z , 4**z > and let S be the <u>open</u> paraboloid $z = 9 - (x^2 + y^2)$ & $z \ge 5$.
Use Divergence Theorem to find $\iint_S \boldsymbol{F.n}\ \boldsymbol{d\sigma}$ (where n is the outer normal vector to our surface)

2') Let **F = < 4x, 8y , 4**z > and let S be the cylinder $x^2 + y^2 = 4$ which is closed on top through z =3; but open on bottom through the plane x+2y+z=2.
Use Divergence Theorem to find $\iint_S \boldsymbol{F.n}\ \boldsymbol{d\sigma}$ (where n is the outer normal vector to our surface)

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3) Use <u>**Stokes' Theorem**</u> to find the outward flux of CURL(F)
$\iint_S \mathbf{Curl}(\boldsymbol{F}).\boldsymbol{n}\ \boldsymbol{d\sigma}$ of the vector field $\mathbf{F} = < 6\mathbf{y},\ 4\mathbf{x},\ \mathbf{z} >$ across the upper part of the sphere $x^2 + y^2 + z^2 = 25$ whose boundary C lies on the plane z = 4.

4) Let **F = < 5y, 8x, z** > and let S be the <u>open</u> paraboloid $z = 10 - (x^2 + y^2)$ & $z \ge 6$ with boundary C. Find the counter clockwise circulation $\oint_C F.dr$

(i) directly (by parametrizing C), and

(ii) by Stokes' Theorem.

(iii) Find $\iint_S \mathbf{Curl}(\boldsymbol{F}).\boldsymbol{n}\ \boldsymbol{d\sigma}$ by Stokes' Theorem, and

(iv) solve part (iii) directly.

4') Use Stokes' theorem to evaluate the counterclockwise circulation of the field **F** = $y^2\boldsymbol{i} - y\boldsymbol{j} + 3z^2\boldsymbol{k}$ around the boundary of the <u>ellipse</u> in which the plane $2x + 6y - 3z = 6$ meets the cylinder $x^2 + y^2 = 1$ (counterclockwise).

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5) Solve the IVP $xyy' - (x^2 + xe^{5x})y^{9/4} = 2y^2 \quad ; y(1) = ...$

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6) Change the following (non-exact) D.E <u>**to exact then STOP**</u>!

$$(8 - 7\mathbf{y} + \mathbf{x}^3\mathbf{e}^x)\ \mathbf{dx} + \mathbf{xdy} = 0$$

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7) Solve the DE $(7y^2 + 2xy)y' = (3x^2 + 4xy)$

8) **Solve** the IVP $xy' = y\sin(xy + 5)$; y(3)=0

9) Solve $y' = (\cos x + 3x^2)e^{(x^3 + \sin x)}\ (y^2 - 9)$